THE GROUPING OF ELECTRIC CELLS

A TREATISE

ON THE GROUPING OF ELECTRIC CELLS

BY

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1 Illustration

New Impression

London
E. & F. N. SPON, Limited, 57 HAYMARKET

New York
SPON & CHAMBERLAIN, 123 LIBERTY STREET

1914

PREFACE

THE chief special features of this treatise on the grouping of cells, are as follows:—

(1) A demonstration that the rule hitherto taught as a perfectly accurate one for grouping cells so as to get the greatest possible current with a given external resistance, cannot even be relied upon to give the best *regular* group (Chap. II.). An example is mentioned in which a regular group that is not in accordance with this rule, gives 400 times the current obtainable with the one that is.

(2) A demonstration that the current obtainable with the best regular group may be less than that given by some irregular group (Chap. III.). Irregular groups have hitherto been ignored, presumably having been regarded as self-evidently bad.

(3) A really accurate rule for grouping cells so as to get the greatest possible current (Chap. III.).

(4) A rule for finding the smallest number of cells that will send a given current through a given external resistance (Chap. V.).

It is assumed that the reader is already acquainted with the elements of electricity and of algebra.

W. F. D.

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PART I.—The Grouping of Similar Cells for Greatest Current.

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THE

GROUPING OF ELECTRIC CELLS

ERRATA

Page 23, last line, *for* -2(3)- *read* -1(6)-

Page 24, last line, *for* or *read* for

Page 27, line 9, *for* seven *read* one

Page 28, line 5, *insert* + *between* 2X *and* 1

Page 40, Exercise 18, *for* -10(2)-1(1)- *read* -1(10)-1(11)-

Page 49, Exercise 23, *for* 88 *read* 87, *and for* 3-ohms *read* 3 ohms

Dunton's "Grouping of Electric Cells"

cally as in Fig. 1, where the unequal parallel lines represent the plates of the cell, the space between them representing the medium (usually a liquid) through which the current passes from the plate represented by the shorter and thicker line, to that represented by the longer and thinner line. The horizontal lines represent the conductors by which the cell is connected to other cells or to the *external circuit*.

THE

GROUPING OF ELECTRIC CELLS

PART I.—The Grouping of Similar Cells for Greatest Current

CHAPTER I

INTRODUCTORY.

BY similar cells I mean cells having the same electromotive force and the same resistance: except where otherwise stated, all groups are supposed to consist of such cells.

It is customary to represent a cell diagrammatically as in Fig. 1, where the unequal parallel lines represent the plates of the cell, the space between them representing the medium (usually a liquid) through which the current passes from the plate represented by the shorter and thicker line, to that represented by the longer and thinner line. The horizontal lines represent the conductors by which the cell is connected to other cells or to the *external circuit.*

Fig. 2 shows three cells connected *in series*; that is to say, with the positive plate of one connected to the negative plate of another, and so on throughout. As is well known, the electromotive force of such a group is the sum of the electromotive forces of the

FIG. 1.

FIG 2.

cells composing it; so that if the electromotive force of each cell in Fig. 2 is 2 volts, that of the group is 6 volts. As the current passes through the three cells in succession, it is clear that the resistance, like the electromotive force, must be three times that of a single cell; and similarly with other series groups.

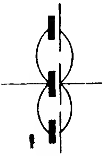

FIG. 3

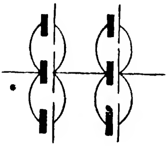

FIG. 4.

Fig. 3 shows three cells connected *in parallel*; that is to say, with the positive plates of all the cells connected to one another, and their negative plates likewise connected to one another. This arrangement gives the current three paths, each equal in con-

ductivity to that offered by a single cell, with the consequence that the resistance of the group is one-third that of a single cell. Its electromotive force, however, is the same as that of a single cell: in fact, the group behaves in every way as if it were one cell with plates three times as wide as those ofthe cells of which it is composed; and similarly with other parallel groups.

Fig. 4 shows two groups like that in Fig. 3, connected together in series. This forms a group which might conveniently be described as "two ranks of three," or more briefly "two threes," and represented by the symbol –2(3)–, which might be called the *mathematical symbol* of the group. We will in future represent all groups by their mathematical symbols, the number inside the brackets showing how many cells are connected in parallel to form a rank, and the number outside showing how many of these ranks are connected in series, while the horizontal lines show that the whole refers to a group of cells. Fig. 2 would thus be represented by –3(1)–, and Fig. 3 by –1(3)–; while –9(5)–2(4)– would represent a group of nine fives connected in series with a group of two fours—a type of *irregular group* whose importance will be seen later on.

From what has already been said, the reader will easily see the reason of the following elementary rules.

RULE 1.—To find the electromotive force of a group, multiply the electromotive force of a single cell by the number of ranks.

For example; to find the electromotive force of –12(5)–2(4)– when that of a single cell is 1·5 volts, add together the numbers outside the brackets, and multiply the result by 1·5.

RULE 2.—To find the resistance of a regular group, divide the resistance of a single cell by the number of cells in a rank (thus getting the resistance of a rank) and multiply the result by the number of ranks: or, when the group is represented by its mathematical symbol, take the number outside the brackets as the numerator of a fraction, and the number inside as the denominator, and multiply by the resistance of a single cell.

For example; to find the resistance of –7(2)– when that of a single cell is 4 ohms, multiply $\frac{7}{2}$ by 4. The resistance of an irregular group like those mentioned above, is of course found by adding together the separate resistances of the regular groups of which it is composed. Thus, when the resistance of a single cell is 3 ohms, that of –4(5)–1(4)– is $3(\frac{4}{5} + \frac{1}{4})$ ohms.

Knowing the electromotive force and the resistance of a group, the current it will send through a given external resistance can be found by Ohm's law,

$$\text{Current} = \frac{\text{Electromotive force}}{\text{Resistance}}$$

The denominator of this fraction will of course be the *total* resistance, that is to say, the resistance of the group plus that of the external circuit.

Exercise 1.—What current will be sent through an external resistance of 10 ohms by -9(3)-1(2)-, each cell having an electromotive force of 2 volts and a resistance of 3 ohms?

Here the electromotive force of the group is 20 volts, and its resistance, $3(\frac{9}{3} + \frac{1}{2})$, or $10\frac{1}{2}$ ohms. The current is therefore—

$$\frac{20}{10\frac{1}{2} + 10}$$

or about 0·976 ampere.

Exercise 2.—What current will be sent through an external resistance of 22 ohms by 4 1-volt and 2-ohm cells all in series? Answer, about 0·133 ampere.

Exercise 3.—What current will -8(2)- send through an external resistance of 16 ohms, with 2-volt and 4-ohm cells? Answer, $\frac{1}{2}$ ampere.

Exercise 4.—What is the external resistance when 5 1-volt and 1-ohm cells, all in parallel, give a current of 2 amperes? Answer, 0·3 ohm (for, by Ohm's law,

$$\text{Resistance} = \frac{\text{Electromotive force}}{\text{Current}}$$

and the resistance of the external circuit is obtained by deducting the resistance of the group from the total resistance as found by this equation).

EXERCISE 5.—What is the resistance of each cell when a current of 2 amperes is sent through an external resistance of 3 ohms by –9(3)–, the electromotive force of each cell being 2 volts. Answer, 2 ohms (for, proceeding as in exercise 4, we find the resistance of the group, which we know is $\frac{2}{3}$ the resistance of each cell).

EXERCISE 6.—What current will be sent through an external resistance of 6 ohms by –1(2)–1(1)–, with $1\frac{1}{2}$-volt and 4-ohm cells? Answer, $\frac{1}{4}$ ampere.

SYMBOLS.

In addition to the mathematical group symbols already explained, the following will frequently be employed, and are here explained once for all.

C, the number of cells.

E*c*, the electromotive force of each cell.

R, the total resistance.

R*c*, the resistance of each cell.

R*x*, the resistance of the external circuit.

K, the number of ranks.

K*i*, the number of ranks in the *ideal regular group* (see Chap. II.).

CHAPTER II.

REGULAR GROUPS.

THREE cells can only be arranged in two different regular groups, namely $-1(3)-$, and $-3(1)-$. We cannot, unfortunately, arrange them in $-\frac{1}{2}(6)-$, so as to get half the electromotive force and $\frac{1}{12}$th the resistance of a single cell, nor in $-12(\frac{1}{4})-$, so as to get 12 times the electromotive force and 48 times the resistance. If such groups were possible, the rule for finding the best group would be a very simple one, as will presently be seen. (By the *best* group I mean the arrangement of the given cells that will cause them to send the greatest current through the given resistance, without regard to economy, which will not be considered till we get to Chap. IV.)

Referring to the table of symbols at the end of Chap. I., it will be seen that, in any regular group, the number of cells in each rank must be $\frac{C}{K}$. We will for the present assume that K and $\frac{C}{K}$ can be fractional. The resistance of each rank being—

$$Rc \div \frac{C}{K} \text{ or } \frac{Rc.K}{C}$$

the resistance of any regular group is $\frac{Rc.K^2}{C}$. I

and the current—

$$\frac{Ec.K}{\frac{Rc.K^2}{C} + Rx} \quad . \quad . \quad . \quad \text{II}$$

Multiplying and dividing formula II. by $\frac{C}{K}$, we get the equivalent expression—

$$\frac{Ec.C}{Rc.K + \frac{Rx.C}{K}}$$

which, again, is equivalent to—

$$\frac{Ec.C}{\left\{\sqrt{Rc.K} - \sqrt{\frac{Rx.C}{K}}\right\}^2 + 2\sqrt{Rc.Rx.C}} \quad . \text{ II}a$$

From this formula it will be seen that if we start from the point where $\sqrt{Rc.K} - \sqrt{\frac{Rx.C}{K}} = 0$, and make a number of successive increases of any amount whatever, in the value of K, each increase makes $\sqrt{Rc.K}$ greater and $\sqrt{\frac{Rx.C}{K}}$ smaller, thus increasing the denominator of the fraction and reducing the current. And if, starting from the same point, we make a number of successive reductions in the value of K, each reduction makes $\sqrt{Rc.K}$ smaller and

$\sqrt{\frac{Rx.C}{K}}$ greater, but the square of the resulting negative value of $\left\{\sqrt{Rc.K} - \sqrt{\frac{Rx.C}{K}}\right\}$ is of course positive; so that the reductions, like the increases, in the value of K, will increase the denominator of the fraction and reduce the current. Now, the point from which any increase or any reduction in the value of K causes a reduction in the current must be the point where the value of K is such that the current has its greatest possible value; therefore, the regular group that gives the greatest possible current, which we will call the *ideal regular group*, is the one in which $Rc.K = \frac{Rx.C}{K}$, or in which $\frac{Rc.K^2}{C} = Rx$; that is to say, the one whose resistance is equal to the resistance of the external circuit.

Taking Ki to represent the number of ranks in the ideal regular group, we have—

$$\frac{Rc.Ki^2}{C} = Rx \quad . \qquad \text{III}$$

from which we get—

$$Ki = \sqrt{\left(\frac{C.Rx}{Rc}\right)} \qquad \text{IV}$$

Previous writers have taught that the best available regular group is the one whose resistance approximates most closely to that of the external circuit, or, in other words, to that of the ideal regular

group. This is plausible; but, as the following examples will show, it is not true.

Suppose we have 62 1-volt and 4-ohm cells, and an external resistance of 31 ohms. The resistance of -2(31)- would be $\frac{8}{31}$ ohm, and that of -31(2)- would be 62 ohms; that is to say, the resistance of the group with 2 ranks would be between 30 and 31 ohms less than the external resistance, while that of the group with 31 ranks would be exactly 31 ohms more than the external resistance. Therefore, as 62 cells cannot be arranged in a regular group with any intermediate number of ranks, the usual rule would tell us to adopt the group -2(31)-, which would give a current of—

$$\frac{2}{\frac{8}{31} + 31}$$

or about 0·064 ampere. But with -31(2)- we should get a current more than five times as great, namely—

$$\frac{31}{\frac{31 \times 4}{2} + 31}$$

or exactly $\frac{1}{3}$ ampere.

Again, with 1201 1-volt and 1-ohm cells, and an external resistance of 600 ohms, the usual rule would tell us to put the cells all in parallel; for 1201 is a prime number, so that the only possible regular groups are -1(1201)- and -1201(1)-; and the resist-

ance of the former group evidently differs from the external resistance by less than 600 ohms, while that of the latter group differs by more than 600 ohms. But with all the cells in parallel the current is only—

$$\frac{1}{\frac{1}{1201} + 600}$$

or about $\frac{1}{600}$ ampere, while with all in series it is almost exactly *400 times as great*, namely—

$$\frac{1201}{1201 + 600}$$

or about $\frac{2}{3}$ ampere.

The rule that will presently be given in place of this very unsatisfactory one, depends upon the hitherto unrecognised fact that if A, B, C, are any three numbers forming a geometrical progression, and if B is the number of ranks in the ideal regular group, then the regular groups in which the numbers of ranks are respectively A and C, will give exactly the same current as each other. For example: if we have 8 1-volt and 1-ohm cells, and an external resistance of 2 ohms, –2(4)–, –4(2)–, and –8(1)–, answer these conditions; for the resistance of the middle group is equal to that of the external circuit, and the ranks, 2, 4, 8, form a geometrical progression. Therefore, according to the law just stated, –2(4)– should give the same current as –8(1)–; and it will be found

that this is so, the current being in either case exactly 0·8 ampere. The following is a mathematical proof that the law holds good in every case.

Let Ki represent, as before, the number of ranks in the ideal regular group; then, whatever number N represents, $\frac{Ki}{N}$, Ki, $N.Ki$, will form a geometrical progression. With the group in which the number of ranks is $\frac{Ki}{N}$, the current will be (by formula II.)—

$$\frac{\frac{Ec.Ki}{N}}{\frac{Rc}{C}\left(\frac{Ki}{N}\right)^2 + Rr} \quad . \quad . \quad . \quad . \quad . \quad (a)$$

and with that in which the number of ranks is $N.Ki$, the current will be—

$$\frac{Ec.N.Ki}{Rc\,(N.Ki)^2 + Rr} \quad . \quad . \quad . \quad . \quad . \quad (b)$$

These fractions can easily be brought to the same numerator, Ec, in which case the denominators will be respectively—

$$\frac{Rc.Ki}{C.N} + \frac{Rr.N}{Ki} \quad . \quad . \quad . \quad . \quad . \quad (c)$$

and—

$$\frac{Rc.N.Ki}{C} + \frac{Rr}{N \cdot Ki} \quad . \quad . \quad . \quad . \quad . \quad (d)$$

It therefore only remains to prove that $(c) = (d)$, which can be done by assuming that this equation is

true and showing that it is reducible to one that has already been proved. For $(c) = (d)$ if

$$\frac{Re.Ki^2}{C.N} + Rx.N = \frac{Re.Ki^2.N}{C} + \frac{Rx}{N}$$

which is true if—

$$\frac{Re.Ki^2}{C.N} - \frac{Re.Ki^2.N}{C} = \frac{Rx}{N} - Rx.N$$

which, again, is true if—

$$\frac{Re.Ki^2}{C}\left(\frac{1}{N} - N\right) = Rx\left(\frac{1}{N} - N\right)$$

and this is evidently true if the resistance of the ideal regular group is equal to that of the external circuit, which has already been proved.

When A, B, C, form a geometrical progression, we may say that A and C are *geometrically equidistant* from B, and that any number between A and B is *geometrically nearer* to B than C is. We have just seen that if two regular groups have their numbers of ranks geometrically equidistant from that of the ideal regular group, they will give the same current; and we had previously seen that, of two regular groups each having a greater or each having a smaller resistance than the external circuit, the one whose resistance differs least from that of the external circuit gives the greater current. We thus get the following rule :—

RULE 3.—Of the different regular groups that can be formed with a given number of cells, the one that gives the greatest current is that in which the number of ranks is geometrically nearest to the number in the ideal regular group.

Let us apply this rule to the cases in which we have seen the old rule fail so badly.

With 62 1-volt and 4-ohm cells, and an external resistance of 31 ohms, $Ki = \sqrt{\left(\frac{62 \times 31}{4}\right)}$. As usual, there is no need to calculate this square root, for it is evidently the geometrical mean between $\frac{62}{4}$ and 31, and is therefore geometrically nearer to 31 than to 2. Rule 3 thus tells us to adopt the group -31(2)-, by which we get more than five times the current obtainable with the old rule.

With 1201 1-volt and 1-ohm cells, and an external resistance of 600 ohms, $Ki = \sqrt{(1201 \times 600)}$. Here again it is unnecessary to calculate the square root, which is evidently the geometrical mean between 1201 and 600, and therefore geometrically nearer to 1201 than to 1. Our new rule thus tells us to put the cells all in series, by which we get almost exactly 400 times the current obtained with the old rule.

EXERCISE 7.—What regular group of 12 3-ohm

cells will send the greatest current through an external resistance of 4 ohms? Answer, –4(3)–.

EXERCISE 8.—What regular group of 20 2-ohm cells will send the greatest current through an external resistance of 3 ohms? Answer, –5(4)–.

EXERCISE 9.—What regular group of 24 1-ohm cells will send the greatest current through an external resistance of $\frac{1}{3}$ ohm? Answer, –3(8)–.

EXERCISE 10.—What regular group of 6 2-ohm cells will send the greatest current through an external resistance of $\frac{1}{2}$ ohm? Answer, –2(3)–.

CHAPTER III

Irregular Groups, and General Rule.

It seems to have been generally regarded as self-evident that an irregular group (that is to say, a group in which all the ranks are not equal) must be inferior to the best regular group. As a matter of fact, however, it frequently happens that the best group is an irregular one. For example, if we have 38 1-volt and 1-ohm cells, and an external resistance of 1 ohm, the best regular group will be –2(19)–, giving a current of—

$$\frac{2}{\frac{2}{19}+1}$$

or about 1·81 ampere; but with the irregular group, –2(7)–4(6)–, the current would be—

$$\frac{6}{\frac{2}{7}+\frac{2}{3}+1}$$

or about 3·07 amperes.

The fact that we have to consider irregular groups, will of course add to the complexity of our general rule; but not so much as might at first be imagined, or, as I will now prove, the greatest current can

never be obtained from a group in which any rank differs from another by more than one cell.

In any group having one rank of X cells and one of X + Y, with or without other ranks, the resistance of these two ranks is

$$Re\left(\frac{1}{X} + \frac{1}{X + Y}\right), \text{ or } Re\left\{\frac{(X + Y) + X}{X(X + Y)}\right\},$$

and if we take Z cells from the larger rank and add them to the smaller, the resistance becomes—

$$Re\left\{\frac{(X + Y - Z) + (X + Z)}{(X + Z)(X + Y - Z)}\right\},$$

which simplifies to—

$$Re\left\{\frac{(X + Y) + X}{X(X + Y) + Z(Y - Z)}\right\}.$$

This only differs from the preceding fraction in having its denominator increased by $Z(Y - Z)$; so that its value must be smaller in all cases where Y is greater than Z. It is thus clear that the resistance of any group becomes smaller and smaller, as its ranks are made more and more equal; and this reduction of resistance, being unaccompanied by a reduction of electromotive force, results in an increase of current. Therefore, with a fixed number of ranks and a fixed number of cells, the best imaginable group is the regular one; and if the regular group is not possible the best group is that in which no rank differs from another by more than one cell. The "ideal *regular*

group" is thus the "ideal *group*," by which name we will in future refer to it; and an irregular group of the type just described, we will call a *special group*.

In the case of regular groups, we have seen that as the number of ranks differs more and more from Ki (the number of cells remaining constant), the current becomes smaller and smaller. Special groups, however, do not follow this law. it frequently happens that a special group in which the number of ranks is exactly or nearly equal to Ki, is improved by being lengthened or shortened (that is to say, by having the number of its ranks increased or reduced). If, for example, we have 33 1-volt and 1-ohm cells and an external resistance of 12 ohms, $Ki = \sqrt{(396)}$. This is evidently a little less than 20. Multiplying 19 by 20, we see that the geometrical mean between these two numbers is $\sqrt{(380)}$; from which it is clear that $\sqrt{(396)}$ is geometrically nearer to 20 than to 19. With the group of 20 ranks, namely -13(2)-7(1), the current is—

$$\frac{20}{6\cdot5 + 7 + 12}$$

or about 0·784 ampere; but if we shorten this group by three ranks, making it -16(2)-1(1)-, the current becomes—

$$\frac{17}{8 + 1 + 12}$$

or about 0·810 ampere. Thus, though the imaginary regular group of twenty ranks gives more current

than the imaginary regular group of seventeen ranks, the special group of twenty ranks gives considerably less current than the special group of seventeen ranks. This is because the special group of seventeen ranks is much more regular than that of twenty ranks. In the imaginary regular group of seventeen ranks, each rank contains $\frac{33}{17}$ of a cell, and in the corresponding special group, sixteen ranks differ from this in the ratio of $\frac{34}{33}$, and seven in the ratio of $\frac{17}{33}$: but in the imaginary regular group of twenty ranks, each rank contains $\frac{33}{20}$ of a cell; and in the corresponding special group, thirteen ranks differ from this in the ratio of $\frac{40}{33}$, and seven in the ratio of $\frac{20}{33}$. In most cases there is a limit beyond which the lengthening or shortening cannot be carried with advantage, notwithstanding the greater regularity that may thus be obtained; and in some cases no lengthening or shortening is possible without a reduction of current. But, as I will now prove, there is an important class of groups in which lengthening or shortening may always be carried to a certain definite extent with advantage, or at least without disadvantage.

Suppose we have a special group that can be lengthened by having X ranks of X + 1 changed

into X + 1 ranks of X, or shortened by having X + 1 ranks of X changed into X ranks of X + 1. In either case the difference in the number of ranks is 1, and the difference in the resistance is—

$$\frac{Rc\,(X+1)}{X} - \frac{Rc.X}{(X+1)}, \text{ or } \frac{Rc\,(2X+1)}{X(X+1)};$$

the ratio of the difference in the number of ranks to the difference in the resistance, being therefore—

$$\frac{X(X+1)}{Rc\,(2X+1)}.$$

Calling this *the ratio*, the truth of the following propositions will easily be seen: (1) if the ratio is greater than $\frac{K}{R}$ with the original group, the lengthening will increase the value of $\frac{K}{R}$ (and will consequently increase the current, since this is equal to $\frac{K}{R} \times Ec$), and the shortening will reduce it; (2) if the ratio is less than $\frac{K}{R}$ with the original group, the lengthening will reduce the current, and the shortening will increase it; (3) if the ratio is equal to $\frac{K}{R}$ with the original group, neither the lengthening nor the shortening will produce any change in the current; (4) where the lengthening or shortening produces a change in the current, a further lengthening or short-

ening in the same manner will produce a further change in the same direction ; and where the lengthening or shortening leaves the current unchanged, a further lengthening or shortening in the same manner will also leave the current unchanged ; and so on, until no further lengthening or shortening is possible without the introduction of a different size of rank

It follows that the best of the groups that do not contain a smaller rank than X or a larger rank than X + 1, is either that with the greatest possible number of ranks of X, or that with the greatest possible number of ranks of X + 1.

It also follows that the same current can sometimes be obtained from a great number of different arrangements of the same cells. As an instance of this, suppose we have 100 2-volt and 1-ohm cells, and an external resistance of 50 ohms. When the cells are arranged all in series, the value of $\frac{K}{R}$ is evidently $\frac{100}{150}$ or $\frac{2}{3}$, which is also the value of $\frac{X(X+1)}{Rc(2X+1)}$ when X = 1. If, therefore, we turn any number of ranks of 1 into half the number of ranks of 2, we shall leave the value of $\frac{K}{R}$ unchanged ; so that the same current will be obtained whether the cells are grouped in 100 ones, in 50 twos, or in any

of the forty-nine possible groups of ones and twos combined. For example: with -24(2)-52(1)- we get a current of $\frac{152}{12 + 52 + 50}$ or $1\frac{1}{3}$ ampere; and with -4(2)-92(1)- we get a current of $\frac{192}{2 + 92 + 50}$, which is also equal to $1\frac{1}{3}$ ampere.

The statement and discussion of the rule that will presently be given, will be simplified by the use of the following technical terms.

PRELIMINARY GROUP.—The regular or special group in which the number of ranks is the geometrically nearest whole number to Ki.

PRIMARY GROUP.—A regular or special group that does not contain a larger or smaller rank than the preliminary group; the longest and shortest of these groups being called the *long primary group* and the *short primary group* respectively, and the two together, the *extreme primary groups.*

SECONDARY GROUP.—A regular or special group that is not a primary group; those longer than the long primary group being called *long secondary groups,* and those shorter than the short primary group being called *short secondary groups.*

From what has already been proved in this and the preceding chapter, the reader will easily see that the following rule is infallible.

RULE 4.—To find the group that will give the greatest current obtainable with a given number of cells and a given external resistance, proceed as follows:—

(1) Find the best primary group. This is always the extreme primary group with which the value of $\frac{K}{R}$ is the greater, and is the one whose K is geometrically nearer to Ki, if the other is not more regular. If the other is more regular, the values of $\frac{K}{R}$ must be calculated and compared. In the rare cases where the value of $\frac{K}{R}$ is the same in both of the extreme groups, all of the primaries will give the same current. If there is no secondary group more regular than the best primary or geometrically nearer to the ideal (that is to say, having a K that is geometrically nearer to Ki than the K of the best primary group is), the primary will be the group required.

(2) If it is not evident from (1) that the best primary is the best group, see whether the value of $\frac{K}{R}$ is greater with some secondary group, in which case the secondary that gives the greatest

value of $\frac{K}{R}$ is the group required. This part of the rule will not involve much trouble if it is remembered that no group can be better than another unless it is either more regular or geometrically nearer to the ideal.

The following are examples of the application of this rule:—

Suppose we have 20 2-ohm cells and an external resistance of 7 ohms. $\therefore Ki = \sqrt{\left(\frac{20 \times 7}{2}\right)} = \sqrt{(10 \times 7)}$; so that the preliminary group must consist of twos and threes, the extreme primaries being therefore -10(2)-, and -6(3)-1(2)-. But since 10 and 7 are geometrically equidistant from Ki, and since the group of 10 ranks is more regular than the group of 7, the group of 10 must be the best primary. And as no secondary can be more regular than this primary or geometrically nearer to the ideal, the best primary is in this case the best group.

Suppose we have 25 2-ohm cells and an external resistance of 24 ohms. $Ki = \sqrt{\left(\frac{25 \times 24}{2}\right)}$, which is the geometrical mean between 12 and 25. The preliminary group would therefore consist of ones and twos, the extreme primaries being -12(2)-1(1)-,

and –25(1)–. But the geometrical mean between 12 and 25, is of course geometrically nearer to 13 (which gives an irregular group) than to 25 (which gives a regular group); and it is therefore necessary to calculate and compare the values of $\frac{K}{R}$ with these two groups. With the long primary, the value is $\frac{25}{74}$, and with the short primary, $\frac{13}{38}$, or $\frac{26}{76}$, or $\frac{25+1}{74+2}$, which must be greater than $\frac{25}{74}$, since $\frac{1}{2}$ is obviously greater than $\frac{25}{74}$ (this example affords an illustration of how we may frequently find which is the greater of two complex fractions by an easy mental calculation). The best primary group, then, is –12(2)–1(1)–. If there is a secondary group superior to this, it must be geometrically nearer to the ideal than the primary –25(1)–, for it cannot be more regular. But, as we have seen, Ki is geometrically equidistant from 25 and 12; so that it is impossible for any secondary group to be geometrically nearer to the ideal than the regular primary is. The best primary is therefore the group we require.

Suppose we have 50 1·5-ohm cells and an external resistance of 0·6 ohm. $Ki = \sqrt{\left\{\frac{50 \times 0 \cdot 6}{1 \cdot 5}\right\}}$, or

$\sqrt{(5 \times 4)}$. The possible regular group of five ranks will thus give exactly the same current as the imaginary regular group of four ranks, and as it is impossible to have a more regular group, or one that is geometrically nearer to the ideal, –5(10)– is the group we require.

Suppose we have 9 2-ohm cells and an external resistance of 1·1 ohm. $Ki = \sqrt{\left\{\frac{9 \times 1 \cdot 1}{2}\right\}}$, or very nearly $\sqrt{5}$. It is thus between 2 and 3, and geometrically nearer to 2 (since $2 \times 3 = 6$). The preliminary group is therefore –1(5)–1(4)–, which is at the same time the long and the short primary group, for it can be neither lengthened nor shortened without introducing a different size of rank. If any secondary group gives a greater current, it must be either –1(9)– or –3(3)–. But 1 and 3 are geometrically equidistant from $\sqrt{3}$, so that 3 must be geometrically nearer to Ki, which is very nearly $\sqrt{5}$. The best secondary is therefore –3(3)–, with which the value of $\frac{K}{R}$ is $\frac{3}{2 + 1 \cdot 1}$, or a little less than 1. But with the only primary group, the value is $\frac{2}{0 \cdot 4 + 0 \cdot 5 + 1 \cdot 1}$, or exactly 1. This primary is therefore the group we require.

Suppose we have 8 1·5-ohm cells and an exter-

nal resistance of 2 ohms. $K_i = \sqrt{\left\{\frac{8 \times 2}{1 \cdot 5}\right\}}$ or $\sqrt{\left\{\frac{16}{1 \cdot 5}\right\}}$, which is evidently between $\sqrt{(10)}$ and $\sqrt{(12)}$, and therefore more than 3, but geometrically nearer to 3 than to 4. The preliminary group, then, is -2(3)-1(2)-, which is at the same time the short primary group, the long primary group being -4(2)-. With the short primary, the value of $\frac{K}{R}$ is $\frac{3}{1 + \frac{3}{4} + 2}$, and with the long primary, $\frac{4}{3 + 2}$. Mentally increasing the numerator and the denominator of the former fraction by one-third of their value, we see that the fraction is equal to $\frac{4}{5}$, which is also the value of the fraction belonging to the other extreme primary. The two primaries (there are only two in this case) will therefore give the same current; and the only secondary that could give a greater current would be one containing less than 3 ranks, but geometrically nearer to the ideal than the regular primary is. But 2 and 5 are geometrically equidistant from $\sqrt{(10)}$; so that 2 must be geometrically further from K_i than 4 is, and no secondary can give a greater current than the primaries.

EXERCISE 11.—What is the best group of 1[illegible]

1·5-ohm cells for an external resistance of 1 ohm? Answer, -2(5)-1(6)-.

EXERCISE 12.—What is the best group of 12 1·4-ohm cells for an external resistance of 7 ohms? Answer, -6(2)-.

EXERCISE 13.—What is the best group of 12 1·5-ohm cells for an external resistance of 2 ohms? Answer, -4(3)-.

EXERCISE 14.—What is the best group of 10 1-ohm cells for an external resistance of 1·3 ohm? Answer, -1(4)-2(3)-.

EXERCISE 15.—What is the best group of 21 1·4-ohm cells for an external resistance of 4 ohms? Answer, -7(3)-.

EXERCISE 16.—What is the best group of 25 1-ohm cells for an external resistance of 10 ohms? Answer, -1(1)-12(2)-.

NOTE.—Where a deficiency of 1 or 2 per cent. is immaterial, the best primary group, which can usually be found with very little trouble, may be regarded as the best group.

PART II.—The Economical Grouping of Similar Cells.

CHAPTER IV.

THE SHORTEST GROUP.

ONE of the examples in Chapter III showed that 100 2-volt and 1-ohm cells will send a current of $1\frac{1}{3}$ ampere through an external resistance of 50 ohms, when they are arranged in any number of ranks from fifty to one hundred. The beginner will naturally wish to know whether, in cases of this sort, there is any reason for adopting one group rather than another. There are, as a matter of fact, two reasons for preferring a shorter to a longer group: (1) because it has less resistance, and consequently causes less waste of power in heating the cells; (2) because, although the same current passes through the group, there is, on the average, less current passing through each cell (for two cells in series each take the whole current, while two in parallel each take half, and so on), with the consequence that there is a less rapid wearing away of the materials of the cells. These reasons would of course apply with still

more force if the shorter group gave a smaller current, so long as this smaller current was sufficient for the purpose. In any case, then, the most economical group is the shortest that will give sufficient current; and the following examples will show how this group can generally be found without much difficulty.

Suppose we have 12 2-volt and 1·5-ohm cells, and wish to send a current of not less than 0·6 ampere through an external resistance of 9 ohms. Let us first see whether it is possible to do this.

$$K = \sqrt{\left(\frac{12 \times 9}{1 \cdot 5}\right)}$$

or about $8\frac{1}{2}$ (a rough approximation is all that is required for our present purpose), so that the ideal group would give a current of about—

$$\frac{8\frac{1}{2} \times 2}{9 \times 2}$$

or 0·944 (since the total resistance, with the ideal group, is twice that of the external circuit). Reducing this by one-third would leave us just about the current we require, but if we shorten the *group* by one-third, we not only reduce the electromotive force by one-third, but likewise considerably reduce the resistance; so that the current would still be considerably greater than we require. We therefore try shortening the group by about half, say to four ranks (we must of course shorten it in such a proportion as

to leave a whole number of ranks, for we are now trying to find a group that we can adopt). This gives the group -4(3)-, and a current of $\frac{8}{2 + 9}$, or about 0·727 ampere. We could reduce this current by one-seventh, and still have more than the required 0·6 ampere; so we try shortening the group by one fourth (one-fourth is much more than one-seventh, but we have only four ranks, and cannot reduce them by less than one) This gives the group -3(4)-, and a current of—

$$\frac{6}{1 \cdot 125 + 9},$$

or about 0·59 ampere. The previous group, -4(6)-, is therefore the shortest that will give the required current.

Suppose we have 20 1·5-volt and 2-ohm cells, and wish to send a current of not less than $\frac{1}{3}$ ampere through an external resistance of 14 ohms. Here—

$$Kr = \sqrt{\left(\frac{20 \times 14}{2}\right)}$$

or about 12; so that the ideal group would give a current of $\frac{18}{28}$, or 0·642 ampere. This is nearly twice what we require, and we may therefore shorten the group by considerably more than half. Let us try

four ranks, which gives the group -4(5)-, and a current of—

$$\frac{6}{1 \cdot 6 + 14}$$

or about 0·385 ampere. This is one-sixth greater than we require, and we accordingly try shortening the last group by one-fourth, which gives the group -2(7)-1(6)-, and a current of—

$$\frac{4\frac{1}{2}}{\frac{4}{7} + \frac{1}{3} + 14}$$

or about 0·302 ampere. The shortest group that will give the required current is therefore -4(5)-.

EXERCISE 17.—What is the shortest group of 12 1-volt and 1-ohm cells that will send a current of not less than 0·9 ampere through an external resistance of 3 ohms? Answer, -4(3)-.

EXERCISE 18.—What is the shortest group of 21 1-volt and 1·5-ohm cells that will send a current of not less than 1 ampere through an external resistance of 1·7 ohm? Answer, -10(2)-1(1)-.

EXERCISE 19.—What is the shortest group of 6 2-volt and 1·1-ohm cells that will send a current of not less than 2 amperes through an external resistance of 1 ohm? Answer, -2(3)-.

EXERCISE 20.—What is the shortest group of 50 1-volt and 2-ohm cells that will send a current of not

less than 0·5 ampere through an external resistance of 23·3 ohms? Answer, -10(3)-10(2)-.

EXERCISE 21.—What is the shortest group of 20 1·5-volt and 2-ohm cells that will send a current of not less than 1 ampere through an external resistance of 4·5 ohms? Answer, -5(4)-.

EXERCISE 22.—What is the shortest group of 10 1·4-volt and 1-ohm cells that will send a current of not less than 1 ampere through an external resistance of 3·75 ohms? Answer, -2(3)-2(2)-.

CHAPTER V.

The Smallest Group.

Another important problem in economical grouping is to find the smallest group (that is to say, the group with the smallest number of cells) that will send a given current through a given external resistance. As the first step towards solving this, let us see how to find the smallest group on the assumption that we can have a fraction of a cell in a rank, and a fraction of a rank in a group: this we will call the *theoretically smallest group*.

Taking A to represent the required current (amperes), Ct the number of cells in the theoretically smallest group, Kt the number of its ranks, Ft the number of its files (that is to say, the number within the brackets in the mathematical symbol of the group), and other symbols as in the table at the end of Chapter I., we have

$$A = \frac{Kt.Ec}{2Rx}$$

(for the theoretically smallest group must be the ideal arrangement of its cells, otherwise the same current could be obtained from the ideal arrangement of a smaller number of cells). From this equation we get—

$$Kt = \frac{2A \, . \, Rr}{Er} \quad . \quad . \quad . \quad V$$

But we know, from formula IV. (Chapter III.), that—

$$Kt^2 = \frac{Ct \, . \, Rr}{Re} \, ;$$

and dividing each side of this equation by Kt, we get—

$$Kt = \frac{\frac{Ct}{Kt} \times Rr}{Re}$$

And $\frac{Ct}{Kt}$ is, of course, the number of files in the group; so that—

$$Ft = \frac{Kt \, . \, Re}{Rr} = \frac{2A \, . \, Re}{Er} \quad . \quad . \quad . \quad VI$$

It should be noticed that $\frac{2A}{Er}$ is a factor in both Kt and Ft; so that if we first find the value of this expression, we need only multiply it by Rr to get Kt, and by Re to get Ft.

As an easy example of the application of these formulæ, suppose we want to find the smallest number of 2-volt and 1·5-ohm cells that will send a current of not less than 4 amperes through an external resistance of 3 ohms.

$$Kt = \frac{2 \times 4 \times 3}{2} = 12, \text{ and } Ft = \frac{2 \times 4 \times 1 \cdot 5}{2} = 6.$$

The theoretically smallest group is therefore a possible

one, namely $-12(6)-$, and the smallest number of cells is 72.

In practice, however, we should not often find Kt and Ft both whole numbers, and should, therefore, usually require more than Ct cells. Suppose, for example, that the theoretically smallest group has been found to be $-\frac{15}{2}(17)-$. If we increase this fractional Kt by adding half a rank, so as to get the geometrically nearest whole number, we increase both the electromotive force and the resistance of the group by $\frac{1}{15}$, and the *total* resistance by $\frac{1}{30}$. This addition will therefore give a greater current than we want. To make the group give exactly the required current the total resistance must be further increased by $\frac{1}{30}$ of its original amount, so that the original electromotive force and the original total resistance shall each be increased by $\frac{1}{15}$. But $\frac{1}{30}$ of the original total resistance is $\frac{1}{15}$ of the resistance of the original group, or $\frac{1}{16}$ of the resistance of the altered group; and adding $\frac{1}{16}$ to the resistance of a group, means making its resistance $\frac{17}{16}$ of its present amount, which we can

do by employing $\frac{16}{17}$ of its present number of files, since the resistance of a regular group of a given number of ranks varies inversely as the number of its files.

To check the accuracy of this reasoning, let us calculate the current obtained with the theoretically smallest group $-\frac{15}{2}(17)-$, and that obtained with the other group $-8(16)-$. With the theoretically smallest group—

$$A = \frac{\frac{15}{2}\,\mathrm{E}c}{\left(\frac{15}{2} \div 17\right)\mathrm{R}c \times 2}$$

and with the other group—

$$A = \frac{8\,\mathrm{E}c}{\left(\frac{1}{2} + \left\{\frac{15}{2} \div 17\right\}\right)\mathrm{R}c.}$$

Increasing the numerator and denominator of the former fraction by one-fifteenth of their present value we see that the fraction is equal to—

$$\frac{8\,\mathrm{E}c}{(16 \div 17)\,\mathrm{R}c}$$

which (since $\frac{1}{2} = \frac{17}{2} \div 17$) is evidently equal to the fraction belonging to the other group

Let us now apply the same line of argument to the general case where Kt is increased or reduced by X, X being the amount that must be added or subtracted in order to change the fractional Kt into its geometrically nearest whole number. The electromotive force and the resistance of the theoretically smallest group are thus each increased or reduced by $\frac{X}{Kt}$ of their value; and in order that the altered group, though retaining its present number of ranks, shall give the same current as the original group, the resistance must be increased or reduced by a further $\frac{X}{Kt}$ of the resistance of the original group, or by $\frac{X}{Kt \pm X}$ of the resistance of the altered group. This means changing the resistance of the altered group to $\frac{Kt \pm 2X}{Kt \pm X}$ of its present value, which we can do by employing—

$$\frac{Kt \pm X}{Kt \pm 2X}$$

of its present number of files (Ft). The group thus obtained is evidently the theoretically smallest group that has a whole number of ranks (I am assuming that the reader has not forgotten what has been proved in previous chapters). Calling this the *enlarged group* (for it contains a larger number of cells

though perhaps, a smaller number of ranks), and taking Cn to represent the number of its cells, and Kn the number of its ranks, we have—

$$Cn = Kn\left(Ft \times \frac{Kn}{Kn \pm X}\right) = \frac{Ft \,.\, Kn^2}{Kn \pm X} \quad . \quad . \quad \text{VII}$$

(The + must of course be read when the ranks have been increased, and the - when they have been reduced.)

We can now deal with any case that may occur in practice. Suppose, for example, that we want to find the smallest number of 1-volt and 2-ohm cells that will send a current of not less than $\frac{5}{6}$ ampere through an external resistance of 4 ohms

$$Kt = \frac{2 \times \frac{5}{6} \times 4}{1} = \frac{20}{3},\ Ft = \frac{10}{3},$$

and the geometrically nearest whole number to $\frac{20}{3}$ is $\frac{21}{3}$, so that $X = \frac{1}{3}$, $Kn = 7$, and

$$Cn = \frac{10 \times 49}{3 \times \frac{22}{3}} = 22\frac{3}{11}.$$

The group we require must therefore contain at least twenty-three cells. Let us see whether twenty-three will be enough. In cases like this it is seldom

necessary to apply rule 4, for the number of ranks in the enlarged group is almost always the geometrically nearest whole number to the K_r of the nearest whole number of cells to C_n. In the present case, then, we assume that the geometrically nearest whole number to K_r is 7, which gives -2(4)-5(3)- as the preliminary (and short primary) group. The current with this group is—

$$\frac{7\,E_c}{\frac{26}{12}R_c + 4} = \frac{21}{25} = \frac{5\frac{1}{4}}{6\frac{1}{4}}$$

which is obviously a trifle more than the $\frac{5}{6}$ ampere required. The smallest number of cells that will give the required current is therefore twenty-three.

The method just illustrated may be summed up in the following rule:—

RULE 5.—To find the smallest number of cells that will send a given current through a given external resistance, proceed as follows:—

(1) Find the theoretically smallest group, by means of formulæ V. and VI.

(2) If the theoretically smallest group has not a whole number of ranks, find the number of cells in the enlarged group, by means of formula VII.

(3) If the theoretically smallest group that has

a whole number of ranks is not a possible group, see whether the required current can be obtained with the possible group having the same number of ranks and the nearest whole number of cells above the number in this theoretical group. If so, this is the smallest number of cells that will give the required current.

(4) In the exceptional
obtained by (3) does n
current, see (from considerations dealt with in Chap III.) whether this current can be obtained from another arrangement of the same cells; and if not, try one cell more, and so on until the necessary number is found

EXERCISE 23.—What is the smallest number of 1-volt and 5-ohm cells that will send a current of not less than 1·2 ampere through an external resistance of 3-ohms? Answer, 88.

EXERCISE 24.—What is the smallest number of 1-volt and 0·5-ohm cells that will send a current of not less than 3 amperes through an external resistance of 5 ohms? Answer, 90.

EXERCISE 25.—What is the smallest number of 2-volt and 0·6-ohm cells that will send a current of not less than 3 amperes through an external resistance of 2 ohms? Answer, 12.

EXERCISE 26.—What is the smallest number of 2-volt and 0·7-ohm cells that will send a current of not less than 1 ampere through an external resistance of 1 ohm? Answer, 1.

EXERCISE 27.—What is the smallest number of 1-volt and 0·5-ohm cells that will send a current of not less than 4 amperes through an external resistance of 1·2 ohm? Answer, 39.

EXERCISE 28.—What is the smallest number of 1·2-volt and 0·6-ohm cells that will send a current of not less than 2 amperes through an external resistance of 3 ohms? Answer, 20.

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LONDON: PRINTED BY WILLIAM CLOWES AND SONS, LIMITED
GREAT WINDMILL STREET, W., AND DUKE STREET, STAMFORD STREET, S.E.